Robustification of the self-optimizing narrowband interference canceler  
– extremum seeking in complex domain*

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Abstract—Self Optimizing Narrowband Interference Canceller (SONIC) is a novel active disturbance rejection algorithm which combines the gain-freezing technique and automatic adjustment of a complex-valued estimation gain. In the paper a new version of the self-adjustment mechanism, based on the extremum-seeking technique, is proposed. To preserve the compactness of the original controller, the adaptation algorithm is designed to operate in complex domain. The resulting adaptive controller is much more robust than its previous variant.

I. INTRODUCTION

In engineering practice one is often faced with the problem of rejection of narrowband disturbances which usually originate from a machinery containing rotating parts, such as mills, engines, rollers, etc. Their presence in the system may cause a significant drop in the underlying process’ quality. Furthermore, acoustic noise which is a common ‘sideproduct’ of various disturbances, may be dangerous to human health or may lead to a substantial discomfort. For instance, MR devices produce large amounts of narrowband noise which makes communication between a patient and a doctor very difficult [1].

In the low-frequency range, where traditional passive solutions (screens, suspensions, ear protectors, etc.) do not provide a satisfactory rejection capability, active solutions have been applied successfully [2], [3]. The methods belonging to this class rely on the principle of destructive interference – a control signal is generated in such a way that, after passing the plant, its amplitude matches that of the disturbance and its phase is shifted by 180 degrees (Fig. 1).

Self Optimizing Narrowband Interference Canceller (SONIC) is a novel algorithm [4], capable of rejecting narrowband disturbances in the system governed by

\[ y(t) = K_0(q^{-1})u(t-1) + d(t) + v(t) , \]

where \( t \) denotes discrete time, \( q^{-1} \) is the backward shift operator, \( q^{-1}u(t) = u(t-1) \), \( y(t) \) is the system output, \( u(t) \) is the control (cancellation) signal, \( K_0(q^{-1}) \) is a transfer function of a linear stable plant (called secondary path in acoustic applications),

\[ d(t) = a(t)e^{j\omega_0 t} \]

is a narrowband (complex-valued) disturbance with slowly time-varying complex ‘amplitude’ \( a(t) \) and known frequency \( \omega_0 \), while \( v(t) \) is a wideband measurement noise.

SONIC controller may be broken down into two loops. The inner loop predicts the disturbance and generates a suitable control signal. It may be summarized using the following equations

\[
\begin{align*}
\hat{d}(t+1|t) &= e^{j\omega_0}[d(t|t-1) + \hat{\mu}(t)y(t)] \\
u(t) &= -\frac{1}{k_n}\hat{d}(t+1|t) ,
\end{align*}
\]

where \( \hat{d}(t+1|t) \) is the one step ahead prediction of the disturbance, \( \hat{\mu}(t) \) is a small estimation gain and \( k_n = K_n(e^{-j\omega_0}) \) is the so-called nominal, or assumed, gain of the plant at the frequency \( \omega_0 \).

The nominal gain usually differs from the true gain. To prevent degradation of the closed loop performance (or even system instability) caused by the modeling errors, the control loop is augmented with the self-optimization loop

\[
\begin{align*}
z(t) &= e^{j\omega_0}[(1-c_\mu)z(t-1) - \frac{c_\mu}{\hat{\mu}(t-1)}y(t-1)] \\
r(t) &= \rho r(t-1) + |z(t)|^2 \\
\hat{\mu}(t) &= \hat{\mu}(t-1) - \frac{z^*(t)y(t)}{r(t)} ,
\end{align*}
\]

where \( * \) denotes complex conjugation, the term \( z^*(t)y(t) \) is an estimate of the gradient of the cost function

\[
V(t) = \sum_{\tau=0}^{\infty} \rho^\tau |y(t-\tau)|^2 ,
\]

\( c_\mu > 0 \) is a small constant, and \( 0 < \rho < 1 \), is a forgetting constant which determines the range of (local) averaging. The unique and very important feature of SONIC is the fact that estimation gain \( \hat{\mu}(t) \) takes complex values. This allows the controller to cope with the possible gain and phase errors in the plant model [4].

The self-optimization loop was derived assuming that the following steady state approximation holds

\[ K_0(q^{-1})u(t-1) \cong k_0u(t-1) , \]

Fig. 1. Block diagram of a feedback active disturbance rejection system.
where \( k_0 = K_0(e^{-j\omega_0}) \) is the true complex gain of the plant at the frequency \( \omega_0 \).

When the above approximation does not hold, the performance of the controller may suffer. Such a situation may occur, for example, when the amplitude of the control signal varies rapidly due to the high rate of amplitude changes of the disturbance signal \( d(t) \). The approximation (4) may also work poorly in the presence of nonlinearities or a very long transport delay, typical of acoustic plants.

The purpose of this paper is to robustify SONIC against the above mentioned factors by applying the extremum seeking approach in the self-optimization loop. Rather than relying on a certain set of assumptions about the plant, the extremum seeking algorithm senses the gradient of the objective function by introducing small changes to controller parameters and evaluating the effects of this action on the cost function. This results in very good robustness of the resulting adaptive controller [5].

Whereas the adjustable gain of the SONIC controller is a complex number, the classical extremum seeking algorithms perform optimization in the real domain. Obviously, one can apply the extremum seeking principle to SONIC by taking advantage of the real-imaginary decomposition of complex numbers. However, doing so would lose the conciseness offered by the complex calculus. Therefore we propose an extremum seeking algorithm which performs optimization directly in the complex domain.

The paper is organized as follows. Section 2 introduces the proposed extremum seeking scheme. Convergence analysis of the algorithm is done in Section 2 as well. Section 3 presents simulation results, and Section 4 concludes.

II. EXTREMUM SEEKING IN COMPLEX DOMAIN

A. Overview

Fig. 2 shows the block diagram of the proposed extremum seeking algorithm. A ‘probing signal’, in the form of a complex sinusoid (cisoid) with amplitude \( \alpha \) and frequency \( \omega_p \), is introduced into the system by varying the adjustable estimation gain \( \hat{\mu}(t) \). This action causes variations in the instantaneous value of the cost function, which is to be minimized.

To find out the proper direction of gain adjustment, the signal \( V[\hat{\mu}(t)] \) is highpass filtered (to remove the DC offset) and then demodulated. The phase shift \( \phi_0 = \arg G(e^{-j\omega_p}) \), is needed to account for the phase delay introduced by the highpass filter. Finally, \( \hat{\mu}(t) \) is corrected in the proper direction using the integrator.

In case of SONIC, the ‘process under optimization’ block consists of the inner loop of the SONIC controller, the unknown plant and the cost function

\[
V[\hat{\mu}(t)] = |y(t;\hat{\mu}(t-1))|^2,
\]

where \( y(t;\hat{\mu}) \) denotes the process that ‘settles down’ in a closed loop for a given value of \( \hat{\mu} \).

Note that our choice of the cost function implies that, unlike the original SONIC, we will be dealing with the deterministic case, i.e. \( v(t) \equiv 0 \). However, our simulation evidence shows that the proposed algorithm works well in the stochastic case, too (see Section 4 for details).

B. Convergence analysis

Observe that the cost function \( V[\mu] \) is real-valued and therefore non-analytic. This means that the classical complex calculus cannot be used to analyze the properties of the proposed controller. The symbolic calculus which is applicable in this case was introduced by Wirtinger. Wirtinger’s derivatives are defined as [6]

\[
\frac{\partial}{\partial \mu} = \frac{1}{2} \left[ \frac{\partial}{\partial \text{Re} \mu} - j \frac{\partial}{\partial \text{Im} \mu} \right], \quad \frac{\partial}{\partial \mu^*} = \frac{1}{2} \left[ \frac{\partial}{\partial \text{Re} \mu} + j \frac{\partial}{\partial \text{Im} \mu} \right].
\]

Denote by \( \mu_o \) the optimal (unknown) value of the estimation gain. If the function \( V(\mu) \) is sufficiently smooth, it may be approximated around \( \mu_o \) as \(^1\)

\[
V(\mu) \approx V_o + \frac{\partial V}{\partial \mu} \Delta \mu + \frac{\partial^2 V}{\partial \mu^2} (\Delta \mu)^2 + \frac{\partial^2 V}{\partial \mu^2} |\Delta \mu|^2 + 2 \frac{\partial^2 V}{\partial \mu \partial \mu^*} \Delta \mu \mu^* + 2 \frac{\partial^2 V}{\partial \mu \partial \mu^*} |\Delta \mu|^2 |\Delta \mu^*|^2 ,
\]

where \( V_o = V(\mu_o) \), \( \Delta \mu = \mu - \mu_o \), and all derivitives appearing on the right-hand side of (6) are evaluated at \( \mu = \mu_o \).

Since \( \mu_o \) is a stationary point of \( V(\mu) \), it follows that the first order derivatives \( \partial V/\partial \mu \) and \( \partial V/\partial \mu^* \) vanish. Furthermore, because \( \mu_o \) minimizes the real valued function \( V(\mu) \), the quantity

\[
\frac{\partial^2 V}{\partial \mu^2} (\Delta \mu)^2 + \frac{\partial^2 V}{\partial \mu^*} (\Delta \mu^*)^2 + 2 \frac{\partial^2 V}{\partial \mu \partial \mu^*} |\Delta \mu|^2 |\Delta \mu^*|^2
\]

is real-valued and positive for any nonzero \( \Delta \mu \). Note that \( \partial^2 V/\partial \mu \partial \mu^* \) is a positive real quantity and that \( \partial^2 V/\partial \mu \partial \mu^* > |\partial^2 V/\partial \mu \partial \mu^*|^2 \).

Let

\[
\hat{\mu}(t) = \hat{\mu}(t) + a_p e^{j\omega_p t}
\]

\(^1\)It is worth noticing that if large deviations of \( \mu \) are assumed and/or higher order derivatives are taken into account, the analysis of the algorithm leads to the same conclusion, although it becomes much more tedious.
where \( \hat{\mu}(t) \) denotes the output of the integrator (see Fig. 2) and \( \Delta \hat{\mu}(t) = \hat{\mu}(t) - \mu_o \). If the instantaneous value of \( \mu \) changes according to (7), the cost function varies as

\[
V[\hat{\mu}(t)] \approx V + \frac{\partial^2 V}{\partial \mu^2} \left[ (\Delta \hat{\mu}(t-1))^2 + 2\Delta \hat{\mu}(t-1)\alpha_p e^{j\omega_p(t-1)} + \alpha_p e^{2j\omega_p(t-1)} \right] + \frac{\partial^2 V}{\partial (\mu^*)^2} \left[ (\Delta \hat{\mu}^*(t-1))^2 + 2\Delta \hat{\mu}^*(t-1)\alpha_p e^{-j\omega_p(t-1)} + \alpha_p e^{-2j\omega_p(t-1)} \right] + 2\frac{\partial^2 V}{\partial \mu \partial \mu^*} \left[ (\Delta \hat{\mu}(t-1))^2 + a_p^2 + \Delta \hat{\mu}^*(t-1)\alpha_p e^{j\omega_p(t-1)} + \Delta \hat{\mu}^*(t-1)\alpha_p e^{-j\omega_p(t-1)} \right].
\] (8)

where the time indices equal to \( t-1 \) stem from the one-step delay introduced by the plant (see Fig. 2).

The first step in the processing of the signal \( V[\hat{\mu}(t)] \) is the high-pass filtration. The filter \( G(e^{-j\omega_p}) \), \( G(1) = 0 \), removes unwanted constant and slowly changing time-varying components, such as \( V \), \( \frac{\partial^2 V}{\partial \mu^2} \left[ (\Delta \hat{\mu}(t))^2 \right] \), \( \frac{\partial^2 V}{\partial \mu \partial \mu^*} \left[ (\Delta \hat{\mu}^*(t))^2 \right] \), and \( \frac{\partial^2 V}{\partial \mu \partial \mu^*} \left[ (\Delta \hat{\mu}(t))^2 \right] \). Hence, the output of the high-pass filter (see Fig. 2) can be approximately written down in the form

\[
h(t) \approx a_p \frac{\partial^2 V}{\partial \mu^2} \left[ 2G(e^{-j\omega_p})\Delta \hat{\mu}(t-1)e^{j\omega_p(t-1)} + G(e^{-2j\omega_p})e^{2j\omega_p(t-1)} \right] + a_p \frac{\partial^2 V}{\partial (\mu^*)^2} \left[ 2G(e^{j\omega_p})\Delta \hat{\mu}^*(t-1)e^{-j\omega_p(t-1)} + G(e^{2j\omega_p})e^{-2j\omega_p(t-1)} \right] + 2a_p \frac{\partial^2 V}{\partial \mu \partial \mu^*} G(e^{j\omega_p})\Delta \hat{\mu}^*(t-1)e^{j\omega_p(t-1)} + G(e^{2j\omega_p})e^{j\omega_p(t-1)} \right].
\] (9)

Note that there are some differences between the fidelity of the steady state approximations done in (4) and (9). In case of (4), the quality of the approximation depends upon the bandwidth of the control signal \( u(t) \). However, in order to attain high cancellation quality, the properties of the control signal must reflect the properties of the disturbance. Hence the quality of (4) may be poor in certain cases, e.g. when the amplitude of the disturbance varies rapidly. On the other hand, the quality of (9) depends on the rate at which \( \Delta \hat{\mu}(t-1) \) changes, i.e., on the adaptation speed, which can be made arbitrarily slow (although, when the optimal settings vary with time, this could result in some performance degradation).

The difference equation that governs \( \hat{\mu}(t) \) takes the form (see Fig. 2)

\[
\hat{\mu}(t) = \hat{\mu}(t-1) - \alpha \xi(t),
\] (10)

where \( \alpha > 0 \) is the adaptation gain and

\[
\xi(t) = h(t)a_p e^{j\omega_p(t-1)} + \delta \phi_0.
\]

Employing the fact that \( \Delta \hat{\mu}(t) = \hat{\mu}(t) - \mu_o \), one arrives at the following relationship which describes evolution of the tracking error

\[
\Delta \hat{\mu}(t) = \Delta \hat{\mu}(t-1) - \alpha \Delta \hat{\mu}^*(t-1) + G(e^{-j\omega_p})\Delta \hat{\mu}^*(t-1)e^{j\omega_p(t-1)} + G(e^{-2j\omega_p})e^{2j\omega_p(t-1)} + G(e^{j\omega_p})\Delta \hat{\mu}(t-1) + G(e^{2j\omega_p})e^{j\omega_p(t-1)}
\] (11)

Observe that the new value of \( \Delta \hat{\mu}(t) \) depends on both \( \Delta \hat{\mu}(t-1) \) and its conjugate. One can check the stability of the difference equation (12) by augmenting it with its conjugate. This leads to the following state space description

\[
x(t) = Ax(t-1) + d(t),
\] (13)

where \( x(t) = [\Delta \hat{\mu}(t) \Delta \hat{\mu}^*(t)]^T \) is the augmented state,

\[
A(t) = \begin{bmatrix}
1 - \gamma - \delta W(t) & -[\gamma W(t) + \delta^* e^{2j\phi_0}] \\
-\gamma W(-t) + \delta e^{-2j\phi_0} & 1 - \gamma^* - \delta^* W(-t)
\end{bmatrix}.
\]

\[
\gamma = 2\alpha a_p^2 \frac{\partial^2 V}{\partial \mu \partial \mu^*} G(e^{j\omega_p})e^{j\phi_0}
\]

\[
\delta = 2\alpha a_p^2 \frac{\partial^2 V}{\partial \mu^2} G(e^{-j\omega_p})e^{j\phi_0}
\]

\[
W(t) = e^{2j\omega_p(t-1)},
\]

and, finally, \( d(t) = [d(t) \ d^*(t)]^T \) where

\[
d(t) = -\alpha a_p^* e^{j\phi_0} \left[ \frac{\partial^2 V}{\partial \mu^2} G(e^{-j\omega_p})e^{j3\omega_p(t-1)} + \frac{\partial^2 V}{\partial (\mu^*)^2} G(e^{-2j\omega_p})e^{-j\omega_p(t-1)} \right].
\]

is a time-varying ‘excitation’.

For sufficiently large values of \( \omega_p \), averaging methods can be applied to analyze the behavior of (13). The dynamics of the averaged system is described by the following equation

\[
\bar{x}(t) = \bar{A}\bar{x}(t-1),
\] (14)
where $\bar{x}(t) = [\Delta \bar{\mu}(t) \Delta \bar{\mu}^*(t)]^T$ is the averaged state and

$$\bar{A} = \begin{bmatrix} 1 - \gamma & -\delta^* e^{j2\phi_0} \\ -\delta e^{-j2\phi_0} & 1 - \gamma^* \end{bmatrix}$$

denotes the state transition matrix of the averaged system.

The averaged system is stable provided that the eigenvalues of the matrix $\bar{A}$ lie inside the unit circle. Applying the Jury’s stability criterion [7] to the characteristic polynomial of $\bar{A}$

$$\det(\bar{A} - \lambda I) = \lambda^2 - 2\lambda \text{Re}(1 - \gamma) + |1 - \gamma|^2 - |\delta|^2$$

one can show that this takes place when

$$-1 < |1 - \gamma|^2 - |\delta|^2 < 1,$$

$$[2\text{Re}(1 - \gamma)]^2 < [2\text{Re}(1 - \gamma) + |\gamma|^2 - |\delta|^2]^2.$$  \(15\)

Since it holds that $\partial^2 V/\partial \mu \mu^* > |\partial^2 V/\partial \mu^2|$, both conditions can be satisfied for $\phi = \arg G(e^{-j\omega_p})$ and a sufficiently small value of $\alpha$.

**Remark 1:** Note that the averaged system equation allows for a certain degree of mismatch between the phase delay of the complex sinusoid feeding the mixer and the phase delay introduced by the highpass filter (or other unmodeled dynamics). The sufficient stability condition which may be derived from the first equation of (15)

$$|\gamma|^2 < 2\text{Re}(\gamma) < 2$$

shows that, for the small values of adaptation gain $\alpha$, a mismatch of up to nearly $90^\circ$ may be tolerated. This is very beneficial for enhancing the robustness properties of the proposed controller.

**Remark 2:** Quite unexpectedly, the mixing process must be performed using the signal $e^{j[\omega_p(t-1)+\phi_0]}$. Although one could probably expect that using $e^{-j[\omega_p(t-1)+\phi_0]}$ would do the job as well, it is not the case. Indeed, when the latter signal is used, the eigenvalues of the averaged system matrix

$$\bar{A}' = \begin{bmatrix} 1 - \delta e^{-j2\phi_0} & -\gamma^* \\ -\gamma & 1 - \delta^* e^{j2\phi_0} \end{bmatrix}$$

are unstable for any values of $\phi_0$, $a_p$, and $\alpha$. The results of simulation experiments support this observation.

**Remark 3:** The choices of probing frequency and highpass filter transfer function are important steps in practical implementation of the extremum seeking algorithm. Based on simulations and real-world experiences we can put forward the following guidelines: 1. In accordance with the above analysis, the probing frequency shouldn’t be too low as this could adversely affect the convergence of the adaptation process. 2. The probing frequency shouldn’t be too high as then the unmodeled dynamics of the plant comes into play. Vaguely speaking, the changes of the adapted parameter are reflected in the cost function with some (generally poorly known) delay. This limits the upper bound on the probing frequency. 3. The transfer function of the highpass filter should be selected in such a way that the modulation of the cost function caused by the probing signal is not attenuated by the filter.

### III. Simulation Results

Simulation experiments were performed to check the behavior of the new adaptive scheme. The impulse response of the simulated plant used in all the experiments is shown in Fig. 3. Note the presence of long transport delay and complex transient behavior. Both factors make the control of the plant difficult.

#### A. Robustness and convergence to the optimal value

In the first experiment the robustness and convergence properties of the new scheme are compared with those of the original scheme. The narrowband disturbance signal used in the experiment was a complex sinusoid with frequency equal to 200 Hz and amplitude governed by the random walk model. The standard deviation of the driving noise incorporated in the random walk model was equal to $\sigma_w = 3.3 \cdot 10^{-4}$. Additionally, a zero-mean wideband measurement noise with standard deviation $\sigma_v = 0.005$ was simulated.

Although the adopted model of disturbance signal can be criticized as naive and unrealistic, such an approach has several important advantages. First, cancellation of random-walk type disturbance is quite difficult because of its stochastic nature. Second, because the rate at which the disturbance’s amplitude changes is constant, there exists an optimal steady-state value of the estimation gain $\mu$. As shown in [4], when the assumption (4) , used to derive the original SONIC controller, holds and $k_n = k_0$, the optimal value of $\mu_o$ is real-valued and approximately equal to $\sigma_w/\sigma_v = 0.066$.

The true dependence of the steady-state mean-squared cancellation error$^2$ on the estimation gain $\mu$ for real values of $\mu$ and $k_n = k_0$ shown in Fig. 4, was evaluated using simulations. The optimal value of the gain is equal to 0.0062. Note a large difference between the theoretical results, obtained under the assumption that the approximation (4) holds, and those observed in simulations. This shows that (4) is a poor description of the plant’s behavior in this case.

$^2$By cancellation error we mean the residual narrowband signal, $e(t) = d(t) - K_0(q^{-1})u(t-1)$. 

Fig. 3. Impulse response of the simulated plant.
The performance of both the old and the newly proposed versions of SONIC was evaluated. The original variant of SONIC was run with \( k_n = K_0(e^{-j\omega_0}), c_{\mu} = 0.0005 \) and \( \rho = 0.9999 \). The algorithm failed to converge and the closed loop was unstable.

The proposed variant of SONIC was run with \( k_n = K_0(e^{-j\omega_0}), \omega_p = 15 \cdot 10^{-4} \) (which corresponds to 1.9 Hz under 8 kHz sampling), \( \alpha_p = 0.001, G(q^{-1}) = (1 - q^{-1})/(1 - 0.9997q^{-1}), \phi_0 = \arg G(e^{-j\omega_p}) = -0.195 \) [rad] and \( \alpha = 0.01 \). The estimation gain converged in mean to 0.0061 + 0.0014j.

The fact that the extremum seeking algorithm converged to a different value than the one determined analytically can be easily explained. The results shown in Fig. 3 were established for \( \Im \mu = 0 \). However, because (4) holds only approximately, the naive assumption that the optimal gain is real valued is not true. Indeed, for a constant value of \( \mu \) equal to 0.0062 the steady state mean-squared cancellation error was equal to 2.1 \cdot 10^{-5}. For \( \mu_0 = 0.0061 + 0.0004j \), the mean-squared cancellation error decreased to 1.9 \cdot 10^{-5}, i.e. the proposed algorithm found the true optimal estimation gain. Note however, that in case of the adaptive system, the presence of probing signal caused a small performance penalty – the steady-state mean-squared cancellation error was equal to 2.1 \cdot 10^{-5} which is still a very good result.

**B. Transient and steady state performance**

In the next experiment the proposed SONIC controller was used to attenuate a real-valued disturbance signal

\[
d(t) = a(t) \cos(0.05t)
\]

where \( a(t) = 1 + 0.2 \sin(1.88 \cdot 10^{-4}t) \) is the slowly time varying amplitude. Under 8 kHz sampling, the above disturbance would correspond to a 200 Hz nonstationary sinusoid with amplitude modulation period of 1.33 s. A low-intensity wideband measurement noise, with variance equal to \( 10^{-6} \), was also included.

\(^3\)We note, however, that SONIC works satisfactorily after adoption of additional safety measures, such as limiting the magnitude of \( \hat{\mu}(t) \).

The modifications in the control loop (2) required to cope with the real-valued case are minor – only the real part of the original control signal \( u(t) = u_R(t) + u_I(t) \) was used to feed the plant. No changes were introduced into the extremum-seeking optimization loop.

This time the parameters used in the adaptation part took the following values \( k_n = 0.3e^{j\pi/4}K_0(e^{-j\omega_0}), \omega_p = 75 \cdot 10^{-4} \) (which corresponds to 9.5 Hz under 8 kHz sampling), \( \alpha_p = 0.005, G(q^{-1}) = (1 - q^{-1})/(1 - 0.9997q^{-1}) \) (under 8 kHz sampling the corresponding 3 dB cutoff frequency of the filter is approximately 1.3 Hz, well below the probing frequency), \( \phi_0 = -1.05 \) [rad] and \( \alpha = 0.01 \). Note that the nominal model differs from the true one by a factor \( 0.3e^{j\pi/4} \). To avoid erratic startup behavior, the adaptive control mechanism was enabled at the instant \( t = 8000 \), with a delay of 8000 samples. Fig. 5 shows the performance of the proposed controller during the initial phase.

The steady-state output signal shows several interesting features in the spectral domain. Fig. 6 shows power spectral densities of the disturbance and steady-state output, evaluated over 160000 samples. Observe the presence of several spectral peaks in the output signal, caused by the introduction of the harmonic probing signal. Note however, that the overall cancellation performance of the proposed controller, equal to 34.8 dB, was more than acceptable. Listening tests confirmed that the artifacts were almost not audible.

**IV. REAL-WORLD EXPERIMENT**

The modified SONIC controller was tested in a real-world active noise control experiment incorporating an acoustic duct. The 60 Hz sinusoid with 10% amplitude modulation served as a disturbance. The period of the modulation was equal to 4 seconds.

The controller was implemented on a Lytech Professional Audio Development Kit board which features the Texas Instrument TMS6727 signal processor. The lowest sampling rate supported by the board is 16 kHz. Since such a value grossly exceeds the needs of the ANC system, the sampling frequency was reduced to 200 Hz using digital decimation.
and interpolation. The 7-th order elliptic IIR filters were used for antialiasing and reconstruction purposes.

The nominal model of the plant was set to $k_n = 1$, which only enabled convergence during the initial phase. The initial value of the complex gain $\tilde{\mu}(t)$ was $0.2 + j0.0$.

The extremum seeking algorithm, enabled after the initial convergence took place, employed the probing signal with frequency equal to 5 Hz and amplitude $a_p = 0.18$. The transfer function of the highpass filter was $G(q^{-1}) = (1 - q^{-1})/(1 - 0.95q^{-1})$ (The corresponding real-world 3 dB cutoff frequency of the filter is 1.5 Hz).

Fig. 7 shows the error signal during the first 10 seconds of the experiment. Note the presence of the residual noise, caused by modulation of the disturbance signal and the improvement in the cancellation performance after the extremum seeking loop was enabled.

Fig. 8 shows the magnitude and the phase of the complex gain $\tilde{\mu}(t)$.

Fig. 9 shows the power spectral density of the steady-state error signal.

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Fig. 8 shows the magnitude and the phase of the complex gain during the experiment. Note that most of the phase error was compensated during the first second of adaptation.

The power spectral density of the steady-state error signal, depicted in Fig. 9, shows that the primary noise was attenuated to the level of 20 dB above the noise floor. The spectral structure of the residual noise is a rather complex one. However, listening test confirmed that the ‘spurs’ are barely audible.

V. CONCLUSIONS

A new self-optimization loop was proposed for the SONIC controller. The optimization is performed using the extremum seeking technique applied in the complex domain. The resulting controller is more robust than the original one.

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