Self-optimizing narrowband interference canceller
– can reference signal help?

Maciej Niedźwiecki* and Michał Meller*
Faculty of Electronics, Telecommunications and Computer Science
Department of Automatic Control, Gdańsk University of Technology
ul. Narutowicza 11/12, Gdańsk, Poland

ABSTRACT

SONIC (Self-Optimizing Narrowband Interference Canceller) is an acronym of the recently proposed active noise control algorithm with interesting adaptivity and robustness properties. SONIC is a purely feedback controller, capable of rejecting nonstationary sinusoidal disturbances (with time-varying amplitudes and/or frequencies) in the presence of plant (secondary path) uncertainties. We show that even though SONIC can work reliably without access to the reference signal, when frequency of the disturbance is unknown and possibly time-varying the algorithm can take advantage of such an additional source of information. Unlike the classical feedforward solutions, the reference signal is used only to extract information about the instantaneous frequency of the disturbance. The time advantage, available due to the fact that the acoustic delay in the system is larger than the electrical delay, allows one to incorporate in the control loop the smoothed, and hence more accurate, frequency estimates. This increases the attenuation efficiency of SONIC and widens its operating range - the modified algorithm can be safely used in the presence of faster frequency changes.

Keywords: Adaptive control, system identification, disturbance rejection.

1 INTRODUCTION

Adaptive noise cancellers (ANC) are traditionally divided into feedforward ones and feedback ones. A feedforward system relies on successive measurements of the so-called reference signal \( r(t) \) – a signal strongly correlated with the disturbance, measured by the sensor (microphone, accelerometer) placed close to the source of unwanted sound (we will focus here on acoustic applications). Since the acoustic delay \( \tau_{ac} \), i.e., delay with which the soundwave emitted by the source of disturbance reaches the point at which it is supposed to be canceled, is considerably longer than the electrical delay \( \tau_{el} \) with which reference measurements are transmitted to the control unit, the controller has the advantage of knowing the disturbance (or, more precisely, of knowing the signal correlated with the disturbance) before it reaches the cancellation point – see Fig. 1a. The controller itself is an adaptive filter that transforms the reference signal into an “antisound” emitted by the canceling loudspeaker to achieve destructive interference. The FXLMS (Filtered-X Least Mean Squares) algorithm [1], [2] is perhaps the one most frequently used for this purpose.

*{maciekn,michal.meller}@eti.pg.gda.pl
For truly wideband (i.e., “unpredictable”) disturbances, such as white noise, feedforward compensation is the only plausible solution. It works as long as the following causality condition is fulfilled

$$\tau_{ac} \geq \tau_{el} + \tau_{pr}$$

(1)

where $\tau_{ac}$ denotes acoustic delay, $\tau_{el}$ denotes electrical delay, and $\tau_{pr}$ is the processing delay introduced by the controller (the time that lapses from the moment of receiving the reference sample to the moment of emitting the antisound “sample”).

When the disturbance is narrowband, i.e., predictable from its past, causality constraint does not apply. In such a case cancellation can be performed using a feedback controller, i.e., a system that relies entirely on the measurements of an error signal $y(t)$ – see Fig. 1b. An effective adaptive control algorithm based on this principle was recently described in [3], [4]. This algorithm, called SONIC (Self-Optimizing Narrowband Interference Canceller) has several advantages over the classical (e.g. FXLMS-based) solutions – due to its self-optimization property it can cope favorably with both disturbance and plant nonstationarity, it avoids nonidentifiability problems which often arise when estimation is carried out in a closed loop, and it is computationally attractive.

While broadband disturbances can be eliminated only using the feedforward technique, narrowband disturbances can be coped with in either feedforward or feedback control configuration. In this paper we describe a new approach to narrowband disturbance canceling, which combines advantages of feedforward and feedback designs. We show that the hybrid feedforward-feedback SONIC controller has better tracking capabilities and is more robust than its original, purely feedback version.

2 SONIC – AN OVERVIEW

The block diagram of the SONIC canceller is shown in Fig. 2. The algorithm was derived assuming that the error signal $y(t)$ (output of the ANC system) can be written down in the form

$$y(t) = K(q^{-1})u(t - 1) + d(t) + v(t)$$

(2)

where $t = \ldots, -1, 0, 1, \ldots$ denotes normalized (dimensionless) discrete time, $K(q^{-1})$ denotes the unknown transfer function of the secondary path ($q^{-1}$ is the backward shift operator), $d(t)$ denotes a nonstationary narrowband disturbance, $v(t)$ is a wideband measurement noise, and $u(t)$ denotes the input signal generated by the controller. To make the analysis simpler, all signals specified above were assumed to be complex-valued (the algorithm for real-valued signals was presented in [5]).

Furthermore, it was assumed that the nonstationary disturbance is governed by

$$d(t) = \gamma(t)e^{j\phi(t)}, \quad \gamma(t) = a(t)e^{j\varphi_0}, \quad \phi(t) = \sum_{i=1}^{t-1} \omega(i)$$

(3)

where $\omega(t)$ denotes the slowly-varying instantaneous frequency and $a(t)$ is a slowly-varying (real-valued) amplitude. Note that $\gamma(t)$ incorporates initial phase $\varphi_0$ of the cisoid.
SONIC can be summarized as follows:

**self optimization**:

\[
\begin{align*}
  z(t) &= e^{j\hat{\omega}(t)} \left[ (1 - c_{\mu})z(t - 1) - \frac{c_{\mu}}{\hat{\mu}(t - 1)} y(t - 1) \right] \\
  r(t) &= pr(t - 1) + |z(t)|^2 \\
  \hat{\mu}(t) &= \hat{\mu}(t - 1) - \frac{y(t)z^*(t)}{r(t)} \\
  u(t) &= -\frac{\hat{d}(t + 1|t)}{k_n[\hat{\omega}(t)]}
\end{align*}
\]

**predictive control**:

\[
\hat{d}(t + 1|t) = e^{j\hat{\omega}(t)}[\hat{d}(t|t - 1) + \hat{\mu}(t)y(t)]
\]

**frequency estimation**:

\[
\hat{\omega}(t + 1) = \hat{\omega}(t) + \gamma \text{Im} \left[ \frac{\hat{\mu}(t)y(t)}{\hat{d}(t|t - 1)} \right].
\]

The multifrequency version of SONIC was presented in [6].

3 HYBRID SONIC

An obvious advantage of SONIC, typical of all feedback ANC systems, is due to the fact that it does not require deployment of a reference sensor. Such a sensor may be expensive and/or difficult to mount. Additionally, it may introduce an acoustic feedback which deteriorates performance of the ANC system. However, this advantage comes at a price – without access to the reference signal, SONIC is bound to
learn about properties of the disturbance, such as its instantaneous frequency $\omega(t)$, by observing the error signal $y(t)$, i.e., the very signal it is trying to kill. Such an internal “conflict of interests” (things that are good for identification are bad for control and vice versa) is an inherent limitation of many adaptive control systems. Under nonstationary conditions it may result in episodes of turbulent, or even bursting, behavior, not acceptable from the practical viewpoint.

The controller proposed in this paper is based on the observation that it may be worthwhile to replace the feedback estimates $\hat{\omega}(t)$ of the instantaneous frequency with the appropriately delayed feedforward estimates $\hat{\omega}_0(t)$ obtained by means of processing the reference signal $r(t) = d_0(t) + v_0(t)$ (5) where $d_0(t)$ denotes the narrowband signal emitted by the source of disturbance, and $v_0(t)$ denotes measurement noise, independent of $v(t)$, picked up by the reference sensor. Such a hybrid solution, depicted in Fig. 3, has two advantages over the purely feedback design:

1. Reference signal is a non-vanishing source of information about the instantaneous frequency of the disturbance. Additionally, even if the ANC system is switched off, the signal-to-noise ratio is usually much higher at the reference point than at the cancellation point.

2. Since the reference signal is measured ahead of time, estimation of the instantaneous frequency of $d(t)$ can be based not only on past, but also on a certain number of “future” (relative to the local time of the controller) samples of the disturbance. Such noncausal estimates, that incorporate smoothing, are more accurate than their causal counterparts.

The hybrid SONIC algorithm incorporates two loops described below.

### 3.1 Feedforward loop – frequency estimation

Estimation of the instantaneous frequency $\omega_0(t)$ of the nonstationary cisoid $d_0(t)$ can be carried out using the adaptive notch filtering (ANF) algorithm given below

\[
\begin{align*}
\varepsilon(t) &= r(t) - \hat{d}_0(t|t-1) \\
\hat{d}_0(t+1|t) &= e^{j\hat{\omega}_0(t)}[\hat{d}_0(t|t-1) + \mu_0(t)\varepsilon(t)] \\
\hat{\omega}_0(t+1) &= \hat{\omega}_0(t) + \gamma_0 \mu_0 \text{Im} \left[ \frac{y(t)}{d_0(t|t-1)} \right]
\end{align*}
\]

where $\mu_0$ (0 < $\mu_0$ < 1) and $\gamma_0$ (0 < $\gamma_0$ << 1) are small gains determining the rate of amplitude adaptation and frequency adaptation, respectively. Although this algorithm resembles the analogous one incorporated in (4), there is one important difference – the gain $\mu_0$ used in (6) is fixed (time-invariant) and real-valued.
Frequency tracking properties of the ANF algorithm (6) can be analyzed using the approximating linear filter (ALF) technique – the stochastic linearization approach proposed in [7]. Suppose that $d_0(t)$ is a constant-modulus cisoid governed by

$$d_0(t + 1) = e^{j\omega_0(t)}d_0(t), \quad |d_0(t)|^2 = a_0^2, \quad \forall t$$  \hspace{1cm} (7)

and that $v_0(t)$ is a zero-mean circular white noise with variance $\sigma_{v_0}^2$. Using the ALF technique, one can show that

$$\hat{\omega}_0(t) \cong H_1(q^{-1})e(t) + H_2(q^{-1})\omega_0(t)$$  \hspace{1cm} (8)

where $e(t) = \text{Im}[v_0(t)d_0^*(t)/a_0^2]$ denotes real-valued white noise with variance $\sigma_e^2 = \sigma_{v_0}^2/(2a_0^2)$ and

$$H_1(q^{-1}) = \frac{\gamma_0\mu_0(1 - q^{-1})q^{-1}}{D(q^{-1})}, \quad H_2(q^{-1}) = \frac{\gamma_0\mu_0 q^{-2}D(q^{-1})}{D(q^{-1})}$$  \hspace{1cm} (9)

Denote by

$$\bar{\omega}_0(t) = E[\hat{\omega}_0(t)|\omega_0(s), s \leq t] = H_2(q^{-1})\omega_0(t)$$  \hspace{1cm} (10)

the mean path of frequency estimates for a particular frequency trajectory. Since $H_2(q^{-1})$ is a lowpass filter with unity static gain $H_2(1) = 1$, for a slowly-varying instantaneous frequency it holds that

$$E[\bar{\omega}_0(t)|\omega_0(s), s \leq t] \cong \omega_0(t - \tau_{\text{est}})$$  \hspace{1cm} (11)

where

$$\tau_{\text{est}} = - \lim_{\xi \to 0} \frac{d}{d\xi} \left\{ \arg[H_2(e^{-j\xi})] \right\} = \frac{1}{\gamma_0}$$  \hspace{1cm} (12)

denotes a nominal (low-frequency) delay introduced by the filter $H_2(q^{-1})$. According to (11), $\bar{\omega}_0(t)$ can be viewed as an estimate of $\omega_0(t - \tau_{\text{est}})$. Hence, delaying the estimate $\bar{\omega}_0(t)$ by $\tau_{\text{est}}$ samples is the simplest way of obtaining smoothed estimates of the instantaneous frequency $\omega_0(t)$.

We note that $\tau_{\text{est}}$ is the optimal delay, i.e., such a time shift that guarantees minimization of the bias component of the mean-squared frequency estimation error (its variance component is invariant with respect to time shifts). When the admissible decision delay $\tau_d$ is smaller than $\tau_{\text{est}}$, bias reduction is less efficient, but still may be significant – the more the larger the value of $\tau_d$. It doesn’t make sense, though, to increase $\tau_d$ beyond $\tau_{\text{est}}$.

More efficient smoothing can be achieved by means of backward-time filtering of the estimates $\bar{\omega}_0(t)$ – for more details see [8].

### 3.2 Feedback loop – self-optimizing control

Since the reference signal is known ahead of time, the control unit can use the smoothed estimates of the instantaneous frequency $\omega(t)$. First, note that due to the acoustic delay introduced by the primary path, the instantaneous frequency of the disturbance $d(t)$ observed at the cancellation point at instant $t$, can be approximated by the instantaneous frequency of the disturbance $d_0(t)$ observed at the reference point at the instant $t - \tau_{\text{est}}$. This will be symbolically written down as

$$\omega(t) \leftrightarrow \omega_0(t - \tau_{\text{ac}}).$$  \hspace{1cm} (13)

Of course, since the primary path is not a pure delay, this time-shifting property holds only approximately.

Similarly, based on the results presented in the preceding subsection, one arrives at

$$\hat{\omega}_0(t) \leftrightarrow \omega_0(t - \tau_{\text{est}}).$$  \hspace{1cm} (14)

Combining (13) with (14), the smoothed (partially debiased) estimate of $\omega(t)$ can be obtained in the form
\[ \hat{\omega}(t) = \hat{\omega}_0(t - \tau). \]  

(15)

where \( \tau = \max\{\tau_{ac} - \tau_{est}, 0\} \) (for simplicity the processing delay was not included in this evaluation).

Based on (15), the following modified version of the control algorithm can be proposed:

\[
\begin{align*}
    z(t) &= e^{j\hat{\omega}_0(t - \tau)} \left[ (1 - c\mu)z(t - 1) - \frac{c\mu}{\hat{\mu}(t - 1)}y(t - 1) \right] \\
    r(t) &= \rho r(t - 1) + |z(t)|^2 \\
    \hat{\mu}(t) &= \hat{\mu}(t - 1) - \frac{y(t)z^*(t)}{r(t)} \\
    \hat{d}(t + 1|t) &= e^{j\hat{\omega}_0(t - \tau)}[\hat{d}(t|t - 1) + \hat{\mu}(t)y(t)] \\
    u(t) &= -\frac{\hat{d}(t + 1|t)}{k_n[\hat{\omega}_0(t - \tau)]} 
\end{align*}
\]  

(16)

4 SIMULATION RESULTS

To check potential benefits offered by the hybrid approach, a special simulation experiment was performed.

Since all results presented in this paper apply to systems with inputs and outputs described by complex numbers, the generated real-valued signals \( d_0(t), d(t), v_0(t) \) and \( v(t) \) were converted to the complex format by adding zero imaginary parts. For cancellation purposes we used \( u_R(t) \) – the real-part of the complex-valued signal \( u(t) \) worked out by SONIC (a more sophisticated approach to real-valued computations was described in [5]).

The primary and secondary paths were simulated using finite impulse response models of a real acoustic duct. The corresponding impulse responses, shown in Fig. 4, were established under 8 kHz sampling. The primary and secondary delays were equal to 100 samples and 60 samples, respectively, i.e., \( \tau_{ac} = 40 \) Sa.

The primary disturbance \( d_0(t) \), with time-varying amplitude and frequency (see Fig. 5), was generated by filtering a nonstationary sinusoidal signal \( s(t) \)

\[ s(t) = 0.05\sin[\phi(t)], \quad \phi(t) = \phi(t - 1) + \omega(t) \]

using an impulse response of a real acoustic source (established experimentally)

\[ d_0(t) = K_s(q^{-1})s(t). \]  

(17)

The instantaneous angular frequency of \( s(t) \) was governed by

\[ \omega(t) = \omega_\star[1 + 0.1\sin(2\pi t/T)], \quad \omega_\star = 0.05\pi, \quad T \in [40000, 800000] \]
i.e., it was changing sinusoidally around the frequency $\omega_*$. Under 8 kHz sampling this is equivalent to changes around 200 Hz ($\pm 20$ Hz) with the period ranging from 5 seconds to 100 seconds.

Standard deviations of the primary and secondary white measurement noise were identical and equal to $\sigma_v = \sigma_{v_0} = 0.001$ - in the absence of disturbance cancellation the corresponding SNR values ranged between 35 dB and 51 dB at the reference point, and between 26 dB and 40 dB at the cancellation point.

![Figure 5: Example of a primary disturbance $d_0(t)$ used in simulation experiments (generated for $T = 80000$ Sa, which under 8 kHz sampling corresponds to the period of 10 s).](image)

Three approaches were compared: the standard extended SONIC, the proposed hybrid version of SONIC and a special variant of SONIC where the true instantaneous frequency of the disturbance, obtained by delaying the (known) instantaneous frequency of $s(t)$, was sent to the control unit. The latter case, corresponds to the ‘full’ knowledge of disturbance’s instantaneous frequency and serves as a reference.

All variants used identical settings in the self-optimization layer: $c_\mu = 0.0005$ and $\rho = 0.999$. Furthermore, to avoid erratic behavior during initial transients, all algorithms were modified by forcing additional constraints $1 < r(t) \leq 100$ and $0.0005 < |\hat{\mu}(t)| \leq 0.005$. In spite of the fact that the gain of the secondary path changed considerably in the vicinity of $\omega_*$, the nominal gain was in all cases constant and equal to the true gain at the frequency $\omega_*$: $k_0 = K(e^{j\omega_*}) = 1.9 + 0.48j$.

The frequency estimation gain of extended SONIC was set to $\gamma = 0.0025$. Although this value may seem small, it was found that using greater gains resulted in stability problems, caused by too long transport delay in the feedback loop. On the other hand, the frequency estimation mechanism employed in the hybrid version of SONIC could enjoy benefits of higher estimation gains: $\mu_0 = 0.02$, $\gamma_0 = 0.01$. Note that, under such settings, $\tau_{est} = 100$ and the optimal choice of smoothing delay in (15) is $\tau = 0$, i.e. the instantaneous frequency estimates, obtained by means of processing the reference signal, were employed immediately.

The performance of all algorithms was compared using cancellation error, defined as

$$c(t) = d(t) - K(q^{-1})u_R(t-1).$$

The results, depicted in Fig. 6, show that a considerable improvement can be obtained using the hybrid approach. Not only were the cancellation errors reduced by at least one order of magnitude, but also the operating range of the system was widened - the modified algorithm can be safely used in the presence of faster frequency changes. Note that the improved algorithm nearly reaches performance of SONIC with full knowledge of disturbance frequency, which can be regarded as the best possible case.
5 CONCLUSIONS

The problem of suppression of nonstationary narrowband disturbances with time-varying amplitude and frequency was considered and solved using a new control architecture which combines elements of feedforward compensation and feedback control. The resulting hybrid SONIC (Self-Optimizing Narrowband Interference Canceller) yields better performance and is more robust than the purely feedback algorithm proposed earlier. Additionally, it can be safely used in the presence of faster frequency changes.

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